

Crossing the phantom divide with dissipative normal matter in the Israel-Stewart formalism

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A phantom solution in the framework of the causal Israel-Stewart (IS) formalism is discussed. We assume a late time behavior of the cosmic evolution by considering only one dominant matter fluid with viscosity. In the model it is assumed a bulk viscosity of the form $\xi = \xi_0 \rho^{1/2}$, where ρ is the energy density of the fluid. We evaluate and discuss the behavior of the thermodynamical parameters associated to this solution, like the temperature, rate of entropy, entropy, relaxation time, effective pressure and effective EoS. A discussion about the assumption of near equilibrium of the formalism and the accelerated expansion of the solution is presented. The solution allows to cross the phantom divide without evoking an exotic matter fluid and the effective EoS parameter is always lesser than -1 and time independent. A future singularity (big rip) occurs, but different from the Type I (big rip) solution classified in S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D 71, 063004 (2005), if we consider others thermodynamics parameters like, for example, the effective pressure in the presence of viscosity or the relaxation time.

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I. INTRODUCTION

A late cosmic stages, phantom phase with an EoS $\omega < -1$ [1] is not ruled out by the observational data. A phantom scheme around today implies future singularities [2] and moreover, those singularities could be due to the presence of (bulk)viscosity [3]. If the EoS of the dark energy is assumed phantom then there is a violation of the dominant energy condition (DEC), since $\rho + p < 0$. The energy density grows up to infinity in a finite time, which leads to a big rip [4], [5].

Nevertheless, in the context of matter creation is possible to explain a phantom behavior without the need of invoking phantom scalar fields and any modifications in the gravity theory via dissipative effects [6].

The possibility to cross the phantom divide with non ideal fluids was also derived in the framework of general scalar fields theories. In [7] was found that the phantom crossing of the dark energy described by a general scalar-field Lagrangian is unstable with respect to the cosmological perturbations. This is the case if the dominant scalar field is described by the action without interactions with other energy components throughout kinetic couplings and higher derivatives. It was also proved that for general k-essence models the crossing of the phantom

divide causes infinite growth of quantum perturbations on short scales [8]. Nevertheless, if higher derivatives are add to the action a single scalar field can cross the phantom divide without gradient instabilities, singularities or ghosts. This scalar field corresponds to a velocity potential of an imperfect fluid, and in an expansion around a perfect fluid it can identified terms which correct the pressure in the manner of bulk viscosity [9].

The advantage to take a non perfect fluid is that dissipation within the cosmic fluids allows also a violation of DEC [10] but the dark fluid do not need to be phantom. In this case we have an effective pressure given by

$$p_{eff} = p + \Pi, \quad (1)$$

where $p = \omega \rho > 0$, being p the barotropic pressure, ρ the energy density and $\Pi < 0$ is the viscous pressure. So, we write $p_{eff} = (\omega + \Pi/\rho) \rho = \omega_{eff} \rho$ and ω_{eff} it could become negative and then play the role of dark energy [11]. For instance, if we do $\omega = 0$ (dust) and by considering for Π the model $\Pi = -3\xi(\rho)H$ (see later) we have $p_{eff} = \omega_{eff} \rho$, where $\omega_{eff} = -3[\xi(\rho)H/\rho]$ and here, $\xi(\rho) > 0$ is the bulk viscosity coefficient. In goods beads, it is possible to have phantom evolution driven by viscous dark matter and we do not need dark energy characterized by $\omega < -1$. And another question is if the current ω -observational data can be interpreted as ω_{eff} or whether we can detect directly parameters associated to viscosity [12].

The possibility of explain the accelerated expansion of the universe at late times as an effect of the effective neg-

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ative pressure due to bulk viscosity in the cosmic fluids was first considered in [13], [14]

In the Eckart approach [15], where the bulk viscosity introduces dissipation by only redefining the effective pressure of the cosmic fluid as in Eq.(1), with $\Pi = -3\xi H$, the possibility of crossing the phantom divide has been found in [16]. The magnitude of the viscosity to achieve this crossing using cosmological data was evaluated in [17]. Other investigation in the framework of this approach have explored big rip singularities for various forms of the EoS parameter and the bulk viscosity [18], little rip cosmologies [19], [20]; phantom crossing in modified gravity [21], [22]; and unified dark fluid cosmologies [23], [24]. At the level of background evolution a degeneracy between a phantom cosmological model and Λ CDM scenarios with a component with bulk viscosity was studied in [11]. The inhomogeneous EOS for dark energy, where the dependence from Hubble parameter is included in EOS, has been motivated from the possibility to include a time-dependent bulk viscosity [21]. A study of the phantom crossing via these EoS was realized and confronted with astronomical data in [25], [26]

It is a well known results that the Eckart approach has drawbacks related to causality and stability. Nevertheless, it is a reasonable assumption, for example, at early times, if we are thinking in "short distances" between interacting cosmic components, i.e., almost instantaneous propagation between them. So, a null relaxation time is a good setting. At late times this non-causal approach does not work, obviously, and this is the main reason for considering causal approaches for bulk viscosity (non-null relaxation time). Previous investigations consider the Eckart approach as a first simple approach to study viscous cosmologies. For example, the thermal fluctuations in the very early stage of viscous cosmology and the spectral index and non-gaussianity was studied in [27]. Also, the structure formation in a viscous CDM universe was considered in [28]. The Eckart and causal formalism was used to face the problems at non-linear level of the Λ CDM model in [29], where authors proposed a viscous model with cosmological constant, which not present, in principle, drawbacks, but reject viscous cosmologies as viable framework for unified models of the dark sector.

The investigation on the nature of phantom behavior from dissipative process would not be complete understood without taking into account a more physical approach like the full Israel-Stewart (IS) causal thermodynamics. Since in this framework there is a great difficulty to obtain solutions to the main equations, only some partial results have been found. In the special case where the bulk viscosity coefficient takes the form $\xi(\rho) \sim \rho^{1/2}$, a big rip singularity solution was obtained in this formalism for a late time FRW flat universe filled with only one barotropic fluid with bulk viscosity [30]. Nevertheless, this solution was obtained in the linear IS theory which relies on the assumption of small deviations from thermodynamics equilibrium, i.e., $|\Pi| < p$. This assumption is not hold in the case of accelerated expansion, as the

observed at late times of the cosmic evolution or during inflation.

Our aim in this work is to study the thermodynamical properties of the phantom solution that is also obtained in the IS formalism when a more consistent expression for the relaxation time is taken into account, derived from the speed of bulk perturbations (see [31]) and showed in Eq.(5). In the big rip solution found in [30] the relaxation time τ was defined as ξ/ρ , where ξ is the bulk viscosity coefficient and ρ is the energy density of the matter component. The main result was that the EoS of the barotropic fluid with bulk viscosity must be of phantom type. In other words there is no crossing of the phantom divide due to the viscosity in the full causal formalism, when the above expression for τ is assumed. Nevertheless, in the big rip solution present here the EoS parameter of the fluid is in the range $0 < \omega < 1/2$ and the effective EoS due to the presence of viscosity correspond to a phantom matter. So a crossing of the phantom divide due to the viscosity is allowed. From a theoretical point of view this results seems to show that *the crossing of the phantom divide were somehow related to the need to maintain causality* [32]. A similar result was found using the Lichnerowicz approach to viscosity [33]. So, the main motivation to explore phantom solutions using a causal approach is to investigate the physical viability to cross the phantom divide without invoking phantom fields.

We will explore further this phantom solution evaluating their entropy generation, temperature of the fluid and viscous bulk pressure as a function of the cosmic time. We will also discuss the possibility to extend the classification of singularities given in [2] since the singularities obtained with the inclusion of dissipation must be characterized by also the behavior of other thermodynamic parameters, like the effective pressure and the relaxation time.

The organization of the paper is as follows: In Section II we present a brief revision of the IS formalism and we show the phantom solution found. In Section III we discuss the near equilibrium conditions in the case of accelerated expansion. We also explore the behavior on time of the thermodynamics parameters like the entropy generation, the viscous pressure, the temperature, the entropy and the relaxation time. In Section IV we propose to extend the classification of singularities analyze some aspects of the thermodynamical equilibrium. Section V is devoted to conclusions. $8\pi G = c = 1$ units will be used.

II. ISRAEL-STEWART FORMALISM

In what follows we assume only one fluid as the main component of the universe, which experiment dissipative process during cosmic evolution. This fluid obey a barotropic EoS, $p = \omega\rho$, where p is the barotropic pressure and $0 \leq \omega < 1$. For a flat FLRW universe, the

equation of constraint is

$$\rho = 3H^2. \quad (2)$$

In the IS framework the transport equation for the viscous pressure Π is given by [34]

$$\tau \dot{\Pi} + \left(1 + \frac{1}{2}\tau\Delta\right) \Pi = -3\xi(\rho)H, \quad (3)$$

where "dot" accounts for the derivative with respect to the cosmic time. τ is the relaxation time, $\xi(\rho)$ is the bulk viscosity coefficient which depends on the energy density ρ , H is the Hubble parameter, and Δ is defined by

$$\Delta \equiv 3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}, \quad (4)$$

where T is the barotropic temperature, which takes the form $T = \beta \rho^{\omega/(\omega+1)}$ (Gibbs integrability condition when $p = \omega\rho$) with β being a positive parameter. We also have that [31]

$$\frac{\xi}{(\rho + p)\tau} = c_b^2, \quad (5)$$

where c_b is the speed of bulk viscous perturbations (non-adiabatic contribution to the speed of sound in a dissipative fluid without heat flux or shear viscosity), $c_b^2 = \epsilon(1 - \omega)$ and $0 < \epsilon \leq 1$ ($\epsilon = 1 \Leftrightarrow$ H-theorem, entropy production is non-negative), $\xi = \xi_0 \rho^s$ being ξ_0 a positive constant, if the second law of thermodynamics is respected [35] and can be estimated, for example in the Eckart formalism, from the observational data [36]. s is an arbitrary parameter. So, the relaxation time results to be (and $\epsilon = 1$ from now on)

$$\tau = \frac{1}{1 - \omega^2} \frac{\xi}{\rho} = \frac{\xi_0}{1 - \omega^2} \rho^{s-1}, \quad (6)$$

and, according to (4)

$$\Delta = \frac{3H}{\delta(\omega)} \left(\delta(\omega) - \frac{\dot{H}}{H^2} \right), \quad (7)$$

where we have defined the $\delta(\omega)$ parameter

$$\delta(\omega) \equiv \frac{3}{4} \left(\frac{1 + \omega}{1/2 + \omega} \right). \quad (8)$$

So, for $0 \leq \omega < 1$, $\delta(\omega) > 0$. Using Eq.(6) and Eq.(2) we can write

$$\tau H = \frac{3^{s-1} \xi_0}{1 - \omega^2} H^{2(s-1/2)}, \quad (9)$$

and we see that $\tau \rightarrow 0$ when H increase for $s < 1$, in particular if $s = 1/2$ and if $s < 0$. This idea appears reasonable if $\dot{\Pi}$ does not increase faster than τ going to zero and maintaining finite Δ (for instance, if $\omega_{eff} \sim -1$ and $0 \leq \omega < 1$). Negative potencies appears to be consistent with the observational data in the Eckart's framework, in particular, $s \leq -1/2$ [37].

Finally, by using for p , T and c_b^2 the expressions given before, the equation (3) can be written in the form

$$\tau_* \dot{\Pi} + \Pi = -3 \xi_*(\rho) H \left[1 + \frac{1}{1 - \omega^2} \left(\frac{\Pi}{\rho} \right)^2 \right], \quad (10)$$

where the effective relaxation time τ_* and the effective bulk viscosity ξ_* are, respectively,

$$\tau_* = \frac{\tau}{1 + 3(1 + \omega)\tau H}, \quad (11)$$

and

$$\xi_* = \frac{\xi}{1 + 3(1 + \omega)\tau H} = (1 - \omega^2) \rho \tau_*. \quad (12)$$

Since we are interested in the possibility of phantom solution in the framework of dissipative process, we shall also consider the truncated equation version of Eq. (10), where the near equilibrium condition

$$\left| \frac{\Pi}{\rho} \right| \ll 1, \quad (13)$$

allows to neglect the second term in square brackets in Eq. (10) and then it reduces to

$$\tau_* \dot{\Pi} + \Pi = -3 \xi_*(\rho) H. \quad (14)$$

The Eckart formalism (non causal formalism) [15] comes after to set in (14) $\tau_* = 0$ ($\tau = 0$). We note also that if $\tau H \ll 1$ then $\tau_* \approx \tau$, $\xi_* \approx \xi$ and (14) is reduced to

$$\tau \dot{\Pi} + \Pi = -3 \xi(\rho) H. \quad (15)$$

A. Phantom solution

In order to find a phantom solution for a universe filled with one dominant fluid with positive pressure and viscosity we construct a differential equation for the Hubble parameter. By using the conservation equation

$$\dot{\rho} + 3H[(1 + \omega)\rho + \Pi] = 0, \quad (16)$$

the Eq.(2) and the relation $\xi(\rho) = \xi_0 \rho^s$ and Eq.(3), we can obtain the following differential equation

$$\left[\frac{2}{3(1-\omega^2)} \left(\frac{3(1+\omega)\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \right) - 3 \right] H^{2(s-1/2)} + \frac{1}{3^s \xi_0} \left[1 + \frac{3^{s-1} \xi_0 \Delta H^{2(s-1)}}{2(1-\omega^2)} \right] \left[3(1+\omega) + \frac{2\dot{H}}{H^2} \right] = 0. \quad (17)$$

In this framework a big rip solution was found for $s = 1/2$ in [30], using the following Ansatz

$$H(t) = A(t_s - t)^{-1}. \quad (18)$$

It is easy to verify that for $s = 1/2$ a quadratic equation for $A = \text{const.}$ is obtained when Eq.(18) is introduced in Eq.(17). The solutions of this equation are detailed in the Appendix. On the other hand, and as far as we know, for $s \neq 1/2$ (or $s \leq -1/2$) there is not a phantom solution of this type in the IS formalism.

Before to discuss the properties of the solution obtained we will inspect now the possibility of finding a phantom scheme in the truncated IS formalism, represented through Eq.(14). By doing this, we follow the procedure done before. For $s = 1/2$, we obtain the following equation for the Hubble parameter $\ddot{H} + \alpha H \dot{H} + \beta H^3 = 0$, where α and β are both constants. By replacing the solution $H(t) = B(t_s - t)^{-1}$, we have for B an algebraic equation for which there is not a positive root and then there is not a phantom solution. Doing the same thing with the truncated version of IS formalism represent by (15), we find a phantom solution if $\sqrt{3}\xi_0 > 1 + \omega$. Finally, by using the Eckart scheme we find a phantom solution if $\sqrt{3}\xi_0 < 1 + \omega$. So, the constraint over ξ_0 decides whether there is or not a phantom solution in the considered formalism.

The above results indicates that phantom solution is obtained in the non causal framework of Eckart and in the causal formalism of Israel-Stewart. Nevertheless, being both approaches physically different it is important to investigate further all the aspects involved in the causal framework. It is significant that the truncated version of IS formalism, which assume a near equilibrium process, do not admits a phantom solution like the Ansatz mentioned above.

III. THERMODYNAMICAL PROPERTIES OF THE PHANTOM SOLUTION

As we mentioned in the previous section, in the truncated version of the IS formalism represented by Eq.(14) there is no phantom solution of the type $H(t) = A(t_s - t)^{-1}$. Since in this approach the basic equation is constructed demanding the near equilibrium condition, it is expected that a phantom solution may have problems with the requirement of the condition given in the inequality (13). This point has been already discussed by Maartens [38] in the context of dissipative inflation where the universe is filled by only one ordinary fluid with positive equilibrium pressure and the negative effective pressure ($p_{eff} = p + \Pi$) is due to a viscous stress. For late times behavior the observed acceleration of the expansion is a condition that we expect to obtain from this negative effective pressure, so the condition $\ddot{a} > 0$ leads to

$$-\Pi > p + \frac{\rho}{3}. \quad (19)$$

So the inequality (19) implies that the viscous stress is greater than the equilibrium pressure. The causal approach assume a near equilibrium regime but in order to obtain accelerated expansion the fluid has to be far from equilibrium. To overcome this situation a nonlinear generalization of the causal linear thermodynamics of bulk viscosity has been implemented in [39]. We do not consider here this generalization but we shall show that in the case of the de Sitter solution there is no difference with the results obtained in [39]. In order to explore the thermodynamics behavior of our phantom solution obtained solving the Eq.(17) by means of the Ansatz (18), we will evaluate the viscous pressure, $\Pi(t)$, and the entropy $S(t)$.

The big rip solution obtained for $s = 1/2$ is singular in the sense that only for this value of the s parameter, Eq. (17) do not admit a de Sitter solution. It is straightforward to verify this since that for a de Sitter solution, $H = \text{const.} = H_0$, Eq.(17) becomes in a simple algebraic equation for H_0 , whose solution is given by

$$H_0 = \left\{ \frac{1}{3^s \xi_0} \left(\frac{1 - \omega^2}{1/2 - \omega} \right) \right\}^{1/2(s-1/2)}. \quad (20)$$

So, the above equation represents a solution of the Hubble parameter if $s \neq 1/2$ and $0 < \omega < 1/2$. We can explore first how behaves in this case $\Pi(t)$ and $S(t)$. We do not have a future-time singularity and the time derivatives of the Hubble parameter are zero, so the thermodynamics parameters are more easily evaluated.

Let us begin evaluating $\Pi(t)$. Using Eq.(2) in the continuity equation (16) we can write the following expression

for $\Pi(t)$

$$\Pi = -2\dot{H} - 3(1 + \omega)H^2, \quad (21)$$

so introducing in Eq. (21) the solution $H = H_0$ we obtain

$$\Pi_0 = -3(1 + \omega)H_0^2 = \text{const.}, \quad (22)$$

We examine now the entropy generation and the entropy as a function of the cosmic time. The entropy generation can be evaluated from the expression

$$nT \frac{dS}{dt} = -3H\Pi, \quad (23)$$

where n is the number density of particles, which satisfy the continuity equation

$$\dot{n} + 3Hn = 0, \quad (24)$$

whose solution in terms of the scale factor $a(t)$ is

$$n(a) = n_0 (a/a_0)^{-3}. \quad (25)$$

So using Eqs.(23) and (25) and the expression for the temperature given by $T_0 = \beta \rho_0^{\omega/(\omega+1)} = \beta (3H_0^2)^{\omega/(\omega+1)}$, we obtain

$$n(t) = n_0 \exp(-3H_0 t), \quad (26)$$

and then the entropy as a function of time takes the form

$$S(t) = \text{const.} + 3(1 + \omega) \frac{H_0^2}{n_0 T_0} \exp(3H_0 t). \quad (27)$$

Summarizing, de Sitter solution obtained from a causal dissipative approach can be obtained with a constant viscous pressure with an exponentially increase of entropy. It is interesting to note that the relaxation time τ is also constant. According to Eq. (6)

$$\tau_0 = \frac{\xi_0}{1 - \omega^2} \rho_0^{s-1} = \frac{\xi_0}{1 - \omega^2} (3H_0^2)^{s-1} = \text{const.}, \quad (28)$$

The above results has no differences with the obtained in [39], in the framework of a nonlinear generalization of the causal linear thermodynamics of bulk viscosity. As it is expect on simple argumentes, the effective EoS defined by

$$\omega_{eff} = \frac{p(t) + \Pi(t)}{\rho(t)}, \quad (29)$$

is always equal to -1 and independent of EoS parameter ω .

Let us evaluate now the corresponding parameters of our big rip solution $H(t) = A(t_s - t)^{-1}$, which presents a singularity in its parameters in a finite time. A simple integration gives us the scale factor as a function of time

$$a/a_0 = (t_s - t_0)^A / (t_s - t)^A, \quad (30)$$

so the number density of particles yields

$$n(t) = n_0 \left(\frac{t_s - t}{t_s - t_0} \right)^{3A}. \quad (31)$$

Of course, at the time $t = t_s$ the size of the universe becomes infinite and the number density of particles goes to zero. The temperature is given by

$$T(t) = \beta (3A^2)^{\omega/(\omega+1)} (t_s - t)^{-2\omega/(\omega+1)}, \quad (32)$$

and the viscous pressure can be obtained introducing our Ansatz in Eq.(21) which yields

$$\Pi(t) = -[2 + 3(1 + \omega)A] A (t_s - t)^{-2}, \quad (33)$$

so the viscous pressure becomes infinity at the singularity increasing the temperature of the fluid to infinity, as it can be seen from Eq. (32) since the power $-2\omega/(\omega+1)$ is always negative for $\omega > 0$.

Let us make some comment about the inequality (19) which is the condition to have an accelerated expansion. The energy density of the phantom fluid takes the form

$$\rho(t) = 3A^2 (t_s - t)^{-2}, \quad (34)$$

so the pressure is $p(t) = \omega\rho(t)$. Then introducing Eq.(33) and Eq.(34) together with the expression for $p(t)$ in the inequality (19) it is straightforward to see that the only condition that this inequality imposes is of $A > 0$, which is a condition of our solution.

The increasing rate of entropy can be evaluated from Eq.(23) and we obtain the following expression

$$\frac{dS}{dt} = C (t_s - t)^\eta, \quad (35)$$

where

$$C = \frac{3A^2 [2 + 3(1 + \omega)A]}{n_0 \beta (3A^2)^{\omega/(\omega+1)}} (t_s - t_0)^{3A} > 0, \quad (36)$$

and

$$\eta = \frac{2\omega}{\omega + 1} - 3(1 + A). \quad (37)$$

Since the natural tendency of systems to evolve toward thermodynamical equilibrium is characterized by two properties of its entropy function: $dS/dt > 0$ and $d^2S/dt^2 < 0$ (S is convex), we also evaluate d^2S/dt^2 in order to get new possible constraints on the parameters of the model. Deriving once Eq.(35) we obtain that

$$\frac{d^2S}{dt^2} = C\eta (t_s - t)^{\eta-1} \quad (38)$$

Then for this phantom solution $dS/dt > 0$ and $d^2S/dt^2 < 0$ is satisfied if $\eta < 0$.

Integration of Eq.(35) yields the entropy as a function of the cosmic time

$$S(t) = -const. \left(\frac{C}{\eta+1} \right) (t_s - t)^{\eta+1}, \quad (39)$$

so $S(t) > 0$ if $\eta + 1 < 0$. Then, we have two conditions for η that must be satisfied: $\eta < 0$ and $\eta + 1 < 0$, which reduce to the condition $\eta < -1$, i.e., $\frac{2\omega}{\omega+1} - 3(1+A) < -1$, which leads to the inequality

$$A > \frac{1}{3} \left(1 + \frac{2\omega}{\omega+1} \right) - 1, \quad (40)$$

and since $A > 0$ then (40) is always verified for the values $0 < \omega < 1/2$. So, our solution verifies naturally the thermodynamics requirements: $S > 0$, $dS/dt > 0$ and $d^2S/dt^2 < 0$.

We evaluate the relaxation time introducing Eq.(18) in Eq.(6) for $s = 1/2$, which yields

$$\tau(t) = \frac{\xi_0/\sqrt{3}}{(1-\omega^2)A} (t_s - t). \quad (41)$$

At the singularity the relaxation time goes to zero. A obvious question is if the effective EoS due to the viscosity in a fluid of positive pressure correspond to a phantom matter. Evaluating the effective EoS defined in Eq.(29) we obtain

$$\omega_{eff} = -1 - \frac{2}{3A}, \quad (42)$$

so the effective EoS do not depends on time and is always phantom.

IV. THE SINGULARITY OF THE PHANTOM SOLUTION

The properties of future singularities when the universe is dominated by a phantom fluid with an EoS of the form

$$p = -\rho - f(\rho), \quad (43)$$

was investigated in [2], where a classification in four types was done. The function $f(\rho)$ can be an arbitrary function but the dominant energy condition (DEC) is always violated. It has been pointed out that this EoS may be equivalent to bulk viscosity [10], nevertheless the physical behavior of this EoS in the framework of a perfect phantom fluid and a non phantom fluid with a bulk viscosity in the IS formalism are quite different. In this sense, our solution presents a Type I singularity (Big Rip) if we follow the classification given in [2], where for $t \rightarrow t_s$, $a \rightarrow \infty$, $\rho \rightarrow \infty$, and $|p| \rightarrow \infty$. In our case we have other parameter like the viscous pressure which also can be considered in the characterization of the singularity. For our solution this parameter also diverges at the singularity. But more important is the effective pressure

defined in Eq.(1) since drives the effective EoS of the viscous fluid. Due to non perfect fluids include the effective pressure as a new parameter to be evaluated at the singularity, we propose to extend the classification proposed in [2] in order to taken into account this new behavior for solutions that present singularities in the presence of viscosity. Specifically, we propose to define naturally the Type I^* (Viscous Big Rip) future singularity for non perfect fluids. This singularity can be characterized in the following way: for $t \rightarrow t_s$

$$a \rightarrow \infty, \rho \rightarrow \infty, |p| \rightarrow \infty, |p_{eff}| \rightarrow \infty, \quad (44)$$

and the higher temporal derivatives of H also diverge. Note that p_{eff} in terms of H and \dot{H} is given by the expression

$$p_{eff} = -2\dot{H} - 3H^2, \quad (45)$$

so the divergence of H at the singularity implies directly the divergence of p_{eff} in both thermodynamical approaches of Eckart and IS.

The divergence of the Hubble rate (Eq.18) also implies the divergences of all curvatures. An special feature of our solution is the constancy of the effective EoS given in Eq.(42), which is always phantom. In general, in the framework of perfect fluids obeying the EoS given by Eq.(43) the effective EoS is constant only in some regions like $t \ll t_s$ or $t \sim t_s$ (see for example the solution $H(t) = n(1/t + 1/(t_s - t))$, where n is a positive constant, in [2]). Additionally, note that despite the divergences in the parameters showed in Eq.(44) the relaxation time given by Eq.(41) goes to zero at the singularity. Since this parameter also characterize the thermodynamic behavior of our solution, could be included in the characterization of the singularities.

Other cosmological solutions of universes filled with dissipative fluids, which presents future singularities were found in [3]. In this case the dark energy component was assumed to be a generalized dissipative Chapygin. When the dissipative effects were evaluated in the framework of the non-causal Eckart theory, the barotropic pressure, p , satisfies $|p(t \rightarrow t_s)| \rightarrow 0$, but the effective pressure goes to infinity at the singularity. For the case where the dissipative effects were analyzed in the truncated version of the IS formalism, we obtained solutions with the following behavior for $t \rightarrow t_s$:

$$a \rightarrow \infty, \rho \rightarrow \infty, |p| \rightarrow \infty, |p| \rightarrow constant, \quad (46)$$

where $|p_{eff}| \rightarrow constant$ also, since the bulk pressure Π always satisfy the condition $|\Pi| \ll |p|$. In one of the solutions the constant is zero.

So, these solutions are not included in the classification above suggested and it is an indication that the classification for singularities occurring with dissipative fluids should be extended.

V. FINAL REMARKS

We have discussed in the framework of IS causal formalism the thermodynamics properties of a big rip solution of the type $H = A(t_s - t)^{-1}$. This solution was found for a flat FRW universe filled with a barotropic fluid with an EoS parameter $\omega > 0$. Our solution implies a cosmological scenario where a barotropic fluid with $0 < \omega < 1/2$ behaves like a phantom fluid with constant EoS $\omega < -1$, driving by the viscosity. So, this solution allows to cross the phantom divide with normal matter in the full causal formalism of IS. In the previous phantom solution found in [30] there is no phantom crossing, since the corresponding dissipative fluid must have a phantom EoS from the beginning. In this present solution Eq.(5) was used, which is consistent expression for the relaxation time. Clearly, this is an indication that how the physical scenarios implies in both solutions differ, depending on the expression for the relaxation time. In summary, this solution open the possibility to have effective phantom behaviors without invoking exotic matter in a full causal thermodynamics formalism.

The phantom solution found presents a singularity which leads to infinities in the energy density, pressure and also in the effective pressure, temperature and entropy of the viscous fluid filling up the universe in a finite time in the future. We have argue that the obtained singularity requires to extend the previous classification of singularities realized in [2], in order to include the behavior of the effective pressure which characterize non perfect fluids.

An unexpected behavior of this solution is that the effective EoS is constant and correspond to phantom matter, despite the dependence on time of the thermodynam-

ics parameters.

The accelerated expansion that present this solution implies that the viscous stress is greater than the equilibrium pressure, so the fluid has to be far from equilibrium. This is probably the main criticism to the found solution. One can postulate that the causal thermodynamics holds beyond the near-equilibrium regime, but there are no consistent reasons to do this. Further investigations require to face the consistency of the IS framework for accelerated solutions and the restriction of near equilibrium, assumed in the thermodynamical approaches. One first step in this direction is to explore if a phantom solution of this type may exist in the nonlinear generalization of causal thermodynamics developed in [39]. We will explore this issue in a future work.

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Appendix: A

If $s = 1/2$, Eq.(17) adopts the form

$$\sqrt{3}\xi_0 \left[\frac{2}{3(1-\omega^2)} \left(3(1+\omega) \frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \right) - 3 \right] + \left[1 + \frac{\sqrt{3}\xi_0}{2(1-\omega^2)} \frac{1}{\delta(\omega)} \left(\delta(\omega) - \frac{\dot{H}}{H^2} \right) \right] \left[3(1+\omega) + 2\frac{\dot{H}}{H^2} \right] = 0 \quad (\text{A.1})$$

and this last equation admits a phantom solution. By using the Ansatz given by Eq.(18) in Eq.(A.1), we obtain the following algebraic equation for A

$$\sqrt{3}\xi_0 \left[\frac{2}{3A(1-\omega^2)} \left(3(1+\omega) + \frac{2}{A} \right) - 3 \right] + \left[1 + \frac{\sqrt{3}\xi_0}{2(1-\omega^2)} \frac{1}{\delta(\omega)} \left(\delta(\omega) - \frac{1}{A} \right) \right] \left(3(1+\omega) + \frac{2}{A} \right) = 0, \quad (\text{A.2})$$

and if $0 < \omega < 1/2$ and $\sqrt{3}\xi_0 > (1-\omega^2)/(1/2-\omega)$, one of the solutions in Eq.(A.2) is real and positive.

In Fig.1 we show this positive solution as a function of ω and ξ_0 .

By other hand, according to Eq.(42), in Fig.2 we show $\omega_{eff}(\omega, \xi_0) = -1 - 2/3A(\omega, \xi_0)$ where $A(\omega, \xi_0)$ it is previous positive root.

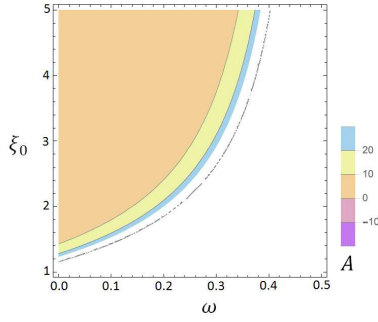


FIG. 1. Positive solution for $A(\omega, \xi_0)$ in the space parameters (ω, ξ_0) .

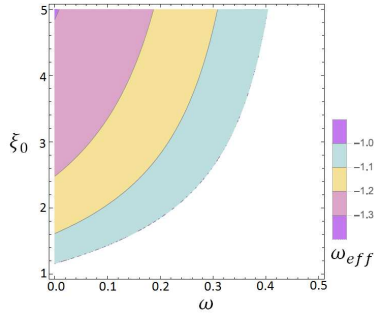


FIG. 2. The effective EoS parameter, ω_{eff} , in the space parameters (ω, ξ_0) .

$$\omega(0) = -1.019^{+0.075}_{-0.080} = \begin{cases} -0.944 \text{ (quintessence zone),} \\ -1.099 \sim -1.1 \text{ (phantom zone).} \end{cases} \quad (\text{A.3})$$

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